Uniform Price Auction for Allocation of Dynamic Cloud Bandwidth

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Abstract—With the ubiquitous adoption of Cloud services by both companies and consumers alike, lack of an efficient system to explicitly price and allocate limited bandwidth has severely impacted the performance of Cloud user-applications. In this context, we consider a two-tier pricing model — consisting of Reserved Phase and Dynamic Phase — that caters to the needs of different kinds of applications. While the Reserved Phase can be used by Cloud users to obtain guarantees on minimum bandwidth well ahead in time, Dynamic Phase can be used to demand and obtain (possibly) additional bandwidth dynamically. Bandwidth being a limited resource, we develop a unique multi-stage uniform price auction with supply uncertainty to dynamically allocate bandwidth to users in the Dynamic Phase. We study the proposed model using a game theoretical approach. Our results prove that proposed auction mechanism is a promising approach for bandwidth allocation. We show that the model promotes the dual advantage of market efficiency and maximum revenue for the Cloud provider. We also demonstrate the price stability using numerical simulations. We argue that for rational, payoff-maximizing tenants of Cloud, the price is stable over the long run which makes the mechanism suitable for practical use.

Keywords—auction, pricing, Cloud, bandwidth, game theory

I. INTRODUCTION

The recent growth of Clouds is mainly driven due to companies shifting from in-house IT infrastructure to utilize third party Cloud service providers, changing consumer preference for Internet-delivered content delivery (Netflix, Hulu, etc. being examples) as well as proliferation of mobile devices. In fact, it has been suggested that with the current trajectory of Cloud growth, computing will emerge as the fifth utility (along with water, electricity, gas and telephone) [7]. However, in the face of increasing demand for Cloud services, limited bandwidth capacity is becoming a critical bottleneck in the future growth of Cloud. Increasing network bandwidth capacity in data-centers is very expensive as it involves the upgrading of network infrastructure. Even with the costly upgrades, it is unlikely that the additional bandwidth will be able to meet the 30% annual growth rate for global data-center traffic [1].

Currently, network bandwidth which is shared by competing tenants, is not explicitly priced and allocated according to the different needs of tenants efficiently. In general, we can classify these demands into two, based on the ability of applications to tolerate interruption in bandwidth obtained over time — (i) interruption tolerant applications, examples being backup tasks, weekly batch jobs, etc; and (ii) interruption intolerant applications, for example Video-on-Demand (VoD), time critical business transactions etc. Interruption intolerant applications require bandwidth to be provisioned for the whole duration of execution to ensure that they are run continuously. The lack of bandwidth guarantees in existing service providers severely impact the performance of many interruption intolerant applications [2]. The conventional standard to allocate Cloud resources is based on the 'Pay-as-you-go' pricing model [3]. This hinges on the critical assumption that Cloud resource provisioning is elastic to provide satisfactory performance [4]. Unlike massive providers like Amazon EC2 and Microsoft Azure, many small to medium size providers are unable to satisfy all requests thus making such scheme not feasible for pricing bandwidth.

In the light of overwhelming demand for Cloud services, inefficiency and limitations in current standards, we need a dynamic pricing model that allocates bandwidth according to the needs of both interruption intolerant and interruption tolerant applications. This will enable the Cloud providers to provide a higher quality of service and serve more tenants despite the resource constraint.

In line with the recent works of two-tier bandwidth allocation model which has been gaining momentum in the research community, we consider a two-tier system consisting of a Reserved Phase and a Dynamic Phase. The Reserved Phase is designed for interruption intolerant applications while the Dynamic Phase caters to both classes of applications. For the interruption intolerant applications, tenants have the knowledge of the minimum bandwidth required to ensure that their applications are not disrupted. This minimum bandwidth is guaranteed for and within the requested duration through the Reserved Phase. Requests in this tier is accepted on a first-come-first-serve basis at a premium price, as the bandwidth is guaranteed, possibly well ahead in time. The premium deters tenants from requesting more than their minimum bandwidth needed. In addition to the minimum bandwidth, tenants (can be expected to) have a good estimate of the additional bandwidth needed which can be acquired through the Dynamic Phase. Besides, interruption tolerant applications choose the Dynamic Phase for requesting bandwidth, instead of going for the Reserved Phase, as their bandwidth demands are more flexible.

In this work, we propose a sealed bid multi-round uniform price auction (described in Section III) to allocate bandwidth to tenants participating in the Dynamic Phase. Here, bandwidths are only allocated for the given time slot (or round). Unallocated bandwidth remaining from the Reserved Phase becomes the supply of available bandwidth in the Dynamic Phase. Interruption tolerant application tenants will only participate in the Dynamic Phase. This work studies the dynamics of the proposed auction model in detail for the Dynamic Phase.
The problem of pricing and allocating pricing Cloud bandwidth has been addressed only recently [8], [12], [11]. So far, these works only consider various forms of pre-determined pricing schemes and none proposed the use of dynamic pricing. Our proposed model is novel as we introduce a multi-round uniform price auction mechanism with supply uncertainty to allocate bandwidth for both interruption tolerant and interruption intolerant applications. Uniform price auction is chosen because of its fairness in charging identical price for identical goods which is a hallmark of a well designed pricing scheme [6].

Different from conventional uniform price auction which has been traditionally used for physical good (stocks, treasury bills, spectrum, land, etc.), we utilize a non-conventional uniform price auction as an allocation mechanism for bandwidth, a transient good. This mechanism has the dual advantage of allowing bandwidth to be shared among more bidders while achieving maximum revenue for the seller at the same time. Our results show that our proposed model provides bidders with incentives to bid truthfully, thereby giving rise to both market efficiency (allocating according to the real needs of the tenants) and maximum revenue possible for the seller without compromising on price stability.

The remainder of the paper is organized as follows. Section III describes our uniform price auction model and introduces the notations used for the rest of the paper. Next, we study the model using game theory in Section IV. In Section V, we perform numerical analysis to study the price stability, before concluding in Section VI.

II. RELATED WORKS

The idea of bandwidth guarantees has been attracting interest from the research community. In the model introduced by Ballani et al. [5], the bandwidth seller provides dedicated minimum bandwidth to each tenant through a simple pricing scheme that are decoupled with the location of the virtual machines (VMs). Building on the works of pricing guaranteed bandwidth, a two-tier probabilistic bandwidth allocation model was proposed in [8] to overcome the inability of tenants to accurately forecast their demands as required in a deterministic model. In this system, bandwidth is guaranteed in the ‘Static’ tier while tenants of the data center have to choose between three fixed pricing policies in the second tier. Comparisons are then made between the deterministic and the proposed probabilistic model.

Making similar observation on the limitations of a deterministic model, a Cloud service model is proposed by Di Niu et al. [11]. Here, a percentage of a tenant’s bandwidth demand is served with guaranteed performance while the rest of its demand is not. Service provider base off historical workload data to predict tenant demands and make actual bandwidth reservations for the tenants. The pricing is based off the requested guaranteed portion and risk undertaken instead of actual allocation.

In another paper by Di Niu et al. [12], the authors examined a free market system where a profit maximizing broker exists between multiple Cloud providers and multiple VoD tenants. The broker buys actual bandwidth from the Cloud providers and in turn sells probabilistic bandwidth guarantees to tenants under certain pricing policy. Through statistical multiplexing, the bandwidth reservations cost is reduced while saving on Cloud resources.

All the above mentioned papers introduced fixed pricing policies in various forms and do not cater for the varying bandwidth demands over time. In addition, the papers by Di Niu et al. are largely based on VoD applications when in fact, Cloud seldom handle only a specific class of applications. The model we proposed takes an innovative approach by introducing an auction mechanism to price bandwidth according to supply and demand without the need to accurately forecast demands. It also takes in account applications that do not require bandwidth guarantees.

Ever since the publication of the classical paper by William Vickrey in 1961 [14], auctions had been extensively studied using the game theoretical approach. Many of the textbook methods focused on studying a two bidders system and often with supply certainty in a single auction which can be found in [10]. We adopt the use of linear allocation function similar to [13] as a starting point for our multi-stage uniform price auction model. Deviating from the simplistic model in [13], we introduce additional degrees of freedom so that our auction model is applicable for bandwidth allocation. First, we enable the bidders to submit independent demand requests from round to round. Second, we introduce an incomplete information market with supply uncertainty where bids are submitted simultaneously. Third, we remove the restriction that bidders are only allowed to submit increasing bid price as the auction progresses. Fourth, we model the behavior of bidding for bandwidth which is a transient good. These features will be further described in Section III and IV.

III. MODEL DESCRIPTION

We define our model for the Dynamic Phase as a multi-stage uniform price sealed bid auction consisting of a monopolistic seller (Cloud provider) and multiple bidders (or tenants of the data-center).

A uniform price auction is one where all units are sold at a market clearing price. As in any auctions, each bidder is to submit a bid consisting of both the quantity of units demanded and the price one is willing to pay per unit. There are two mechanisms to determine the market clearing price and how the goods are allocated. The conventional method is one where the seller allocates the good to the highest bidder first, giving them the full number of units demanded, then the second highest bidder and so forth until the supply of the good is exhausted. The market clearing price is then set to be the lowest winning bid price where demand is equal to the supply. But for our application of bandwidth allocation, we adopt a different mechanism as defined below.

Definition 3.1: In a single round of uniform price auction, allocation functions are formulated based on the bidders’ demand requests and bid prices. The seller then determines the market clearing price and allocates the goods based on these allocation functions. At the end of the auction, all winning bidders pay the market clearing price for their respective allocated quantities.

Instead of *’winners take all’* as in the former mechanism, our model only allocates a certain portion of each winning...
bidder’s demand request to them. Hence, its allows the same supply of bandwidth to be shared among more bidders. At the same time, the seller is able to achieve maximum revenue. Next, we extend this auction to multiple rounds and introduce supply uncertainty.

In our auction model, time is divided into slots and one slot can be assumed to be in tens of minutes or hours. Each slot is equivalent to a bidding round. For bidding round, each bidder is required to submit their bid request specifying the demand quantity and bid price. For bidder $i$ in the $k^{th}$ round, we let the demand quantity be $a_{ki}$ and bid price be $c_{ki}$. From each of the individual bid requests, the seller will formulate the respective allocation function as,

$$q_{ki} = \begin{cases} a_{ki} - b_{ki}(p_k) & p_k < c_{ki} \\ 0 & p_k \geq c_{ki} \end{cases}$$

(1)

where $q_{ki}$ represents the quantity allocated to $i^{th}$ bidder and $p_k$ is the market clearing price charged for per unit of bandwidth at the end of $k^{th}$ round. To simplify the notation, we let $b_{ki} = \frac{a_{ki}}{c_{ki}}$. Here, we can interpret $b_{ki}$ as the tenant’s control variable for bid price. The assumption of a linear allocation function is given in the works by [9].

Prior to the commencement of each bidding round, we assume that tenants have a good estimate on the range of additional bandwidth required with the upper limit being $a_{ki}^{\text{max}}$. This is on top of the minimum bandwidth guaranteed from the Reserved Phase.

Similarly, each bidder has its own independent private valuation of per unit bandwidth, $v_{ki}$. We define $v_{ki}$ as a positive variable for bandwidth allocated that is less than or equal to $a_{ki}^{\text{max}}$ and $v_{ki}$ is zero for $q_{ki} > a_{ki}^{\text{max}}$ because we assume that the tenants have no use for bandwidth allocated in excess of what they required. $v_{ki}$ is history dependent and it will be described in further detail in the next section. Furthermore, since no tenant will submit a bid price higher than one’s own valuation to avoid negative payoff, $\Omega_{ki}$, we have $a_{ki}/b_{ki} \leq v_{ki}$ or equivalently $c_{ki} \leq v_{ki}$.

Lastly, we let $Q_k$ be the bandwidth supply available at round $k$. At the end of each round, the market clearing price, $p_k$ becomes public information while $Q_k$ is only known to the seller at all times.

### IV. Game Formulation

This section studies the proposed auction model for bandwidth allocation using game theory. We determine the tenants’ bidding strategies in order to prove the existence of a unique Nash equilibrium. Subsequently, we derive the conditions for maximum social welfare and examine the efficiency of the model. The focus of this section is to look at how the seller and bidders behave in individual rounds. Thus, for a start, we will treat $v_{ki}$ as a constant before proceeding to extend the model to repeated rounds.

To model the behaviour of the bidders, we let $\Omega_{ki}$ be the the payoff of bidder $i$ in round $k^{th}$,

$$\Omega_{ki} = (v_{ki} - p_k)(a_{ki} - b_{ki}p_k)$$

(2)

Since bidders will either profit or remain status quo by participating in the auction, we have $\Omega_{ki} \geq 0$.

**Proposition 4.1:** It is a weakly dominant strategy for any bidders to submit a demand request with $a_{ki} = a_{ki}^{\text{max}}$.

**Proof:** We look at three possible cases:

(i) $a_{ki} > a_{ki}^{\text{max}}$, (ii) $a_{ki} = a_{ki}^{\text{max}}$, (iii) $a_{ki} < a_{ki}^{\text{max}}$

The first case yields a positive probability of bandwidth over-allocation for some $p_k \in \mathbb{R}^+$, hence resulting in a negative payoff while the latter two cases do not. Next, for players with identical private valuation, $v_{ki}$, and maximum willingness to pay, $a_{ki}^{\text{max}}$, case (ii) yields a higher or equal payoff than case (iii) for all $p_k \in \mathbb{R}^+$ as seen from Eq. (2). For all the possible actions (or strategies) that can be taken by the bidders, submitting a demand request with $a_{ki} = a_{ki}^{\text{max}}$ yields an equal or better payoff than any other actions. Hence, by definition, it is as a weakly dominant strategy.

As seen from Proposition 4.1, bidders will always specify the demand quantity as that of the estimated upper limit, $a_{ki} = a_{ki}^{\text{max}}$, in order to increase its payoff. Similarly, the risk neutral monopolist seller will always set $p_k$ such that revenue is maximized for each and every rounds of $k \in \mathbb{Z}^+$. The seller’s revenue is

$$R_k = p_k \sum_{i=1}^{n} q_{ki} = p_k \sum_{i=1}^{n} a_{ki} - (p_k)^2 \sum_{i=1}^{n} b_{ki}$$

(3)

Maximizing $R_k$,

$$\frac{\partial R_k}{\partial p_k} = \frac{\partial}{\partial p_k}[p_k \sum_{i=1}^{n} a_{ki} - (p_k)^2 \sum_{i=1}^{n} b_{ki}] = 0$$

$$p_k = \frac{\sum_{i=1}^{n} a_{ki}}{2 \sum_{i=1}^{n} b_{ki}}$$

(4)

The price is dependent on the aggregate bandwidth demand, $\sum_{i=1}^{n} a_{ki}$, and aggregate price bids, $\sum_{i=1}^{n} b_{ki}$ submitted by $n$ participating tenants during $k^{th}$ round. However, this may result in negative quantities allocated to those bidders with $p_k > a_{ki}/b_{ki}$. In order to have $q_{ki} \geq 0$ for all $n$ bidders, the seller will have to remove allocation functions with $p_k > a_{ki}/b_{ki}$ and recompute a new $p_k$. Doing so allows the seller to be able to achieve a higher revenue for the round. By substituting Eq. (4) into Eq. (3), the maximum revenue for $k^{th}$ round is

$$R_k^{\text{max}} = \frac{(\sum_{i=1}^{n} a_{ki})^2}{4 \sum_{i=1}^{n} b_{ki}}$$

To prove that seller has the incentive to remove allocation functions with $p_k > a_{ki}/b_{ki}$ for this particular round, we show

$$\frac{(\sum_{i=1}^{n} a_{ki})^2}{4 \sum_{i=1}^{n} b_{ki}} \geq \frac{((\sum_{i=1}^{n} a_{ki}) - a_r)^2}{4((\sum_{i=1}^{n} b_{ki}) - b_r)}$$

where $L$ represents the set of bidders with $p_k > a_{ki}/b_{ki}$ while $a_r$ and $b_r$ denote their aggregate allocation functions. Expanding the equation,

$$\frac{((\sum_{i=1}^{n} a_{ki}) - a_r)^2}{4((\sum_{i=1}^{n} b_{ki}) - b_r)} = \frac{(\sum_{i=1}^{n} a_{ki})^2 - 2a_r \sum_{i=1}^{n} a_{ki} + (a_r)^2}{4((\sum_{i=1}^{n} b_{ki}) - b_r)}$$

$$= \sum_{i=1}^{n} a_{ki}(\sum_{i=1}^{n} a_{ki} - 2a_r) + \frac{(a_r)^2}{4((\sum_{i=1}^{n} b_{ki}) - b_r)}$$

This term is positive
where
\[
\frac{1}{4!} \left[ \sum_{i=1}^{n} a_{ki} \left( \sum_{j=1}^{n} a_{kj} - 2a_j \right) \right] = \frac{(\sum_{i=1}^{n} a_{ki})^2}{4} \frac{1}{\sum_{i=1}^{n} a_{ki}} - b_{ki}
\]
From the new market clearing price, we can derive the total quantity of bandwidth allocated by seller, \( \alpha_k \) in round \( k \).

**Proposition 4.2:** The upper limit of total bandwidth allocated at round \( k \) is 50% of the aggregate demand from \( n \) bidders.

\[
0 < \alpha_k = \frac{\sum_{i=1}^{n} a_{ki}}{2} \leq \frac{\sum_{i=1}^{n} a_{ki}}{2}
\]

**Proof:**
\[
\alpha_k = \sum_{i=1}^{n} \left| q_{ki} \right| = \sum_{i=1}^{n} \left| a_{ki} - b_{ki} \sum_{i=1}^{n} \left| q_{ki} \right| a_{ki} \right| 2 \sum_{i=1}^{n} a_{ki} b_{ki}
\]
Thus, we have
\[
\alpha_k = \frac{\sum_{i=1}^{n} a_{ki}}{2}
\]
The expression is bounded by \( \frac{1}{2} \sum_{i=1}^{n} a_{ki} \) on the upper limit for the case where \( L = \emptyset \).

As the growth of demand for Cloud services outpace the supply of bandwidth available in recent years, it is unlikely that the seller is able to satisfy 50% of the aggregate demand in every round. In view of this, we need to consider the case where \( Q_k < \alpha_k \). The available supply of bandwidth, \( Q_k \), is dependent on the excess bandwidth available from the Reserved Phase. Hence, \( Q_k \), is uncertain and it varies from round to round. This results in \( Q_k \) being the limiting factor in setting the market clearing price.

\[
Q_k = \sum_{i=1}^{n} \left( a_{ki} - b_{ki} p_k^* \right) \Rightarrow p_k^* = \frac{\sum_{i=1}^{n} a_{ki} b_{ki} - Q_k}{\sum_{i=1}^{n} a_{ki}}
\]
where \( p_k^* \) is the market clearing price when \( Q_k < \alpha_k \). Given \( Q_k < \alpha_k \), \( p_k^* > p_k \). This shows that market clearing price increases as the supply of bandwidth available decreases.

**Proposition 4.3:** Bidders will pursue the dominant strategy of submitting a demand request with \( b_{ki} = \frac{a_{ki}}{v_{ki}} \) (or equivalently \( c_{ki} = v_{ki} \)) that is independent of the available bandwidth. This results in a unique Nash equilibrium for round \( k \).

**Proof:** In this non-cooperative game which we assume \( n \) to be large, the rational risk neutral bidders will act to maximize their payoff over the long run. Under these conditions, there exists a unique Nash equilibrium for each round where all bidders will pursue their own dominant strategies. In the following, we examine the two cases of \( Q_k \geq \alpha_k \) and \( Q_k < \alpha_k \).

**Case 1:** \( Q_k \geq \alpha_k \): First, we examine the case of \( Q_k \geq \alpha_k \). Using Eq. (2) and we substitute the allocation function of bidder 1 (assuming \( p_k < a_{k1}/b_{k1} \) and \( i \notin L \)),

\[
\Omega_{k1} = \left| a_{k1} - \frac{\sum_{i=2}^{n-1} a_{ki} + 2a_k}{2(b_k + \sum_{i=2}^{n-1} b_{ki})} \right| a_{k1} - b_{k1}(a_{k1} + \sum_{i=2}^{n-1} a_{ki})
\]
Maximizing \( \Omega_{k1} \),
\[
\frac{\partial \Omega_{k1}}{\partial b_{k1}} = 0
\]
\[
\Rightarrow b_{k1} = \frac{\sum_{i=2}^{n-1} b_{ki}(\sum_{i=2}^{n-1} a_{ki} - 2a_k \sum_{i=2}^{n-1} b_{ki}) + 3a_{ki}}{\sum_{i=2}^{n-1} a_{ki} + 2\sum_{i=2}^{n-1} b_{ki} - a_{ki}}
\]
If we assume \( \sum_{i=2}^{n-1} a_{ki} >> a_{k1} \) (when \( n \) is large) and substitute Eq. (4),
\[
b_{k1} = \frac{(\sum_{i=2}^{n-1} b_{ki})(p_k - v_{k1})}{p_k + v_{k1}}
\]
However, we have \( v_{k1} > p_k \) for the bidder to be allocated positive bandwidth. From Eq. (7), theoretical maximum \( \Omega_{k1} \) can only be achieved when \( b_{k1} < 0 \) which is impractical. To investigate further, in Fig. 1, we plot the general graph of Eq. (7) with constraints \( v_1 > \frac{a_{ki}}{p_k} \), \( a_1 < \sum_{i=2}^{n} a_{ki} \) and \( b_1 < \sum_{i=2}^{n} b_{ki} \).

**Case 2:** \( Q_k < \alpha_k \): Next, we examine the case of \( Q_k < \alpha_k \). Applying the same methodology, constraints and assumptions as above,

\[
\Omega_{k1} = \left| v_{k1} - \frac{a_{k1} + \sum_{i=2}^{n-1} a_{ki} - Q_k}{b_k + \sum_{i=2}^{n-1} b_{ki}} \right|
\times \left| a_{k1} - b_{k1}(a_{k1} + \sum_{i=2}^{n-1} a_{ki} - Q_k) \right|
\]
Maximizing \( \Omega_{k1} \),
\[
\frac{\partial \Omega_{k1}}{\partial b_{k1}} = 0
\]
\[
b_{k1} = \frac{(\sum_{i=2}^{n-1} b_{ki})(p_k - v_{k1})}{p_k + v_{k1}}
\]
For both cases, as seen from Eq. (8) and Eq. (11) which are identical, submitting an allocation function with Eq. (9) yields the practical maximum \( \Omega_{k1} \). Thus, we prove that bidders will pursue the same dominant strategy independent of available
For the case of $Q_k$, it is easy to see that each bidder has the incentive to bid the market clearing price, i.e., $p^*_k$, and the total revenue is $n p^*_k = a_{k_i}^\max$.

By pursuing the dominant strategy as shown by Eq. (9), allocation functions become

$$q_{k_i} = a_{k_i} \left(\frac{v_{k_i} - p_k}{v_{k_i}}\right).$$

This indicates that the maximum price that bidder $i$ is willing to pay is equivalent to its own private valuation of per unit bandwidth, $v_{k_i}$.

**Corollary 4.4:** Tenants bid truthfully under the uniform pricing auction. The absence of bid shading results in an efficient auction where seller is able to achieve the maximum possible revenue while the bidders with greater valuation (those who need it more) are allocated more bandwidth for the same $p_k$.

**Proposition 4.5:** Maximum possible social welfare is achieved if and only if all $n$ participating bidders have $v_{k_i} \geq p_k$ and $Q_k \geq \alpha_k$.

**Proof:** Social welfare in round $k$ is given by

$$\Omega_k = \sum_{i=1}^n v_{k_i} a_{k_i} - 2p_k \sum_{i=1}^n a_{k_i} + (p_k)^2 \sum_{i=1}^n b_{k_i}$$

For the case of $Q_k \geq \alpha_k$,

$$\sum_{i=1}^n \Omega_{k_i} = \sum_{i=1}^n v_{k_i} a_{k_i} - 2p_k \sum_{i=1}^n a_{k_i} + (p_k)^2 \sum_{i=1}^n b_{k_i}$$

$$\Rightarrow \sum_{i=1}^n \Omega_{k_i} \geq \sum_{i=1}^n v_{k_i} a_{k_i} - 2p_k \sum_{i=1}^n a_{k_i} = \sum_{i=1}^n \sum_{j=1}^n a_{k_i} [p_k]$$

For the case of $Q_k < \alpha_k$,

$$\sum_{i=1}^n \Omega_{k_i} = \sum_{i=1}^n v_{k_i} a_{k_i} - \sum_{i=1}^n \sum_{j=1}^n a_{k_i}$$

$$\Rightarrow \sum_{i=1}^n \Omega_{k_i} \geq \sum_{i=1}^n v_{k_i} a_{k_i} - \sum_{i=1}^n \sum_{j=1}^n a_{k_i} = \gamma_i p^*_k$$

Eq. (12) is always greater than Eq. (13). Eq. (12) is maximum when $\sum_{j=1}^n v_{k_i} a_{k_i}$ is largest and $p_k$ is smallest. It occurs when all participating bidders have $v_{k_i} \geq p_k$ or $L = 0$. Achieving maximum possible social welfare also implies that the total bandwidth allocation satisfies the upper limit of 50% of the aggregate demand from all $n$ bidders.

For a tenant who aim to achieve a certain throughput over time, in the case of a video-on-demand provider, $v_{k_i}$ may not be constant from round to round. For this class of bidders, it will value the bandwidth more for the current round if it is not allocated sufficient bandwidth from the previous round. Thus, we model $v_{k_i}$ as a ‘history dependent’ variable based on the concept of Markov Chains. For players which aim to fulfill 50% of $a_{k_i}^\max$ in any round

$$v_{k_i} = 2p_k - p_k - \left[\frac{q_{k-1}}{a_{k_i}^\max\left[\gamma_i p_k\right]}\right]$$

Substituting in Eq. (1) and Eq. (9)

$$v_{k_i} = \frac{3}{2} p_k - \left(\frac{p_k}{v_{k_i}}\right)^2$$

In the general case, we have

$$v_{k_i} = \gamma_i p_k - \left(\frac{p_k}{v_{k_i}}\right)^2$$

For bidders which aim to achieve greater than 50% of $a_{k_i}^\max$, we have $\gamma_i > 3/2$ and $\gamma_i \leq 3/2$ otherwise. The expression which is only dependent on the information available from the previous round behaves like a multi-variable recursive function making the model into a sequential game. Note that even with this expression of $v_{k_i}$, the players’ bidding strategy remains as that of Proposition 4.3.

V. Price Stability Analysis

We perform numerical analysis to study the stability of prices, $p_k$, since equation Eq. (14) involves the use of recursive functions over large number of rounds (or iterations). This allows us to observe how this game evolves over time. To make the simulations more realistic, we introduce $\sigma_i$, which is the maximum budget (or ability to pay) per unit bandwidth. $\sigma_i$ is a constant for player $i$ in all rounds.

We recall that $a_{k_i}^\max$ is the estimated maximum bandwidth that each tenant possibly require for the $k^{th}$ round. However, it is not necessary that they need to be allocated $a_{k_i}^\max$ to satisfy their actual bandwidth needs. Instead, they will bid such as $a_{k_i}^\max γ_i p_k$, where $γ_i$ is the estimated maximum bandwidth requirement for otherwise. The expression $\gamma_i$ as shown in the previous section. Hence, we consider the following three cases:

(i) $\frac{\sum_{i=1}^n v_{k_i}}{n} < 1.5$, (ii) $\frac{\sum_{i=1}^n v_{k_i}}{n} = 1.5$, (iii) $\frac{\sum_{i=1}^n v_{k_i}}{n} > 1.5$

Case (i) occurs when $n$ participating bidders aim to achieve an average of less than 50% of their respective $a_{k_i}^\max$. Case (ii) is for the scenario when bidders want to fulfill an average of 50% of their respective $a_{k_i}^\max$ while Case (iii) happens when they bid to achieve an average of more than 50% of their respective $a_{k_i}^\max$. Case (iii) is the most likely scenario to occur in bandwidth allocation setting as tenants are rational and want to maximize their bandwidth allocation. We present cases (i) and (ii) only for the sake of completeness in our analysis.

To illustrate the results, we simulate an auction of 50 bidders (or n=50) over 1000 rounds repeatedly to make observations on the price stability. We set $a_{k_i}^\max$, $\sigma_i$ and $\gamma_i$ to be random values drawn from independent uniform distributions. We choose a uniform distribution for its randomness to allow us to stress test the model for price stability. Observations are made across different limits for the above distributions. Fig. 2(a), Fig. 2(b) and Fig. 2(c) are generated with the limits for $a_{k_i}^\max$, $\sigma_i$ and $\gamma_i$ set to [1000, 15000], [1000, 2000] and [1.3, 1.7] respectively. Limits of $\gamma_i$ is chosen such that the medium is 1.5. In addition, we set the initial $v_{k_i}$ to be $\sigma_i$, the
maximum budget. In addition, we note that the limits placed on the uniform distribution for numerical simulation purposes only affect the absolute values of $p_k$ and has no effects on its stability. We have also observed that price stability is only affected by $\sum_{i=1}^{n} \gamma_i / n$ and not any other variables. Hence, we will only present the most relevant results with the three cases for different values of $\sum_{i=1}^{n} \gamma_i / n$ due to limited space in the paper.

The results are explained as follows: Fig. 2(a) depicts the general case of $\sum_{i=1}^{n} \gamma_i / n < 1.5$ where the price decreases. Fig. 2(b) depicts the general case of $\sum_{i=1}^{n} \gamma_i / n = 1.5$ where the price converge to a single price point despite that $a_{k,\text{max}}$ varies from round to round for each bidder. This is the equilibrium state where $v_{ki}$ becomes a constant and it is the same for all players. The single price equilibrium can be seen from

$$p_k = \frac{\sum_{i=1}^{n} a_{ki}}{2 \sum_{i=1}^{n} b_{ki}} = \frac{2 \sum_{i=1}^{n} a_{ki}}{\sum_{i=1}^{n} a_{ki}} = 0.5 v_{ki}$$

Fig. 2(b) depicts the general case of $\sum_{i=1}^{n} \gamma_i / n > 1.5$ where the price stabilizes and fluctuates around a certain mean.

Since rational bidders compete to obtain as much bandwidth as possible given their own constraints, the case of $\sum_{i=1}^{n} \gamma_i / n > 1.5$ is the only relevant scenario that we are concerned with. Hence, as observed from Fig. 2c, we can conclude that the price is stable for the proposed bandwidth allocation model.

VI. CONCLUSIONS

This paper considered a two-tier bandwidth allocation model consisting of the Reserved Phase and Dynamic Phase, that caters to most of the applications. We modeled the Dynamic Phase as a multi-round sealed bid uniform price auction. The proposed auction allows bandwidth to be shared among more tenants while allowing the seller to achieve maximum revenue. More importantly, we are able to dynamically price the bandwidth according to actual demand and supply in contrast to the predetermined pricing policies adapted today. We also proved that the tenants bid truthfully under the model, thereby resulting in an efficient market where tenants with higher valuation are allocated more bandwidth. For the realistic case where all participating bidders are rational, we showed that the price is stable.

REFERENCES